REGIONAL SCIENCE ASSOCIATION: PAPERS, XVIII, VIENNA CONGRESS, 1966

THE MULTISECTORAL INTERREGIONAL LONG-RANGE OPTIMAL PLANNING PROBLEM

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In this report we discuss some ways of working out an approximated and aggregated plan for developing the national economy of a country. The purpose of the plan is to provide some general idea of possibilities for future economic development as well as to identify shadow prices of basic resources and products. This approach can be used at the first stage of building up the system of optimal planning and management, a stage which is characterised by insufficient information, by restricted computing capabilities, and by lack of clearness in some theoretical aspects of the development of a planned Socialist economy.

In working out the approximated optimal plan, primary attention should be devoted to:

a) the formulation of a generally correct but rather simple economic development model;

b) the choice of an essentially reasonable but perhaps not exact criterion for optimal economic development;

c) the difficulties of collecting basic information, including experts' estimates and other approximations;

d) the elaboration of more efficient algorithms for approximate solution of special and rather voluminous programming problems.

Let us outline some specific features of the proposed model.

1. A national economy is being dealt with in the form of aggregated territorial complexes (regions). Also some foreign countries or groups of countries which are involved in foreign trade are also considered as separate complexes. These complexes are connected by the most important types of transport media.

2. The model is represented in physical units. Products may be produced in different regions. The same product is regarded as homogeneous in every region, i.e., as having the same consumption properties. A product may be produced by different technological activities. In order to simplify the model, it is assumed that any one activity can yield only one product. (This assumption can be all too easily neglected). The set of activities producing the same product is referred to as an industry, and an activity in a framework of a region is termed an enterprise.

3. If necessary (for instance, when resources are scarce), the upper limits of yearly production are set for particular industries and regions. Such restrictions can also be imposed for the regional economy as a whole (for instance, restrictions on utilization of water resources).

4. Labour resources are evaluated for each region, taking into account

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migration possibilities. The amount of migration is restricted, and its desirable level is found while solving the whole problem. Some more detailed demographic data are taken into account as well. Labour inputs and factors causing migration are considered to be known.

5. One of the most important characteristics of a production enterprise is its rated (nominal) production capacity. But at the same time it is necessary to introduce a certain coefficient which shows the degree to which this capacity can be used due to unrecoverable wear, the extent of technological skill, etc. The coefficient changes with time. The capacity of the transport network is particularly considered in this way.

6. The principal part of the model consists of balance equations of income and expenditure for each product for every region for the last year of the planning period (which is T_0 years). These equations cover: output of the product by different technological activities; current and capital expenditures of the product for the needs of production and transportation; personal consumption; governmental consumption; interregional export and import; foreign trade. The governmental needs are satisfied without fail; others are optimized variables. A linear dependence of current and capital expenditures of products and labour is assumed for every enterprise. For transportation arcs, a piecewise linear dependence is used.

7. Some typical wage levels are singled out, their number being finite. The relative distribution of employees by these levels is known for every industry. Minimal feasible annual income per capita is established for every region.

8. A finite number of personal consumption levels is envisaged. Every level is characterized by a family consumption matrix, for various groups of family income for an average family size. The higher the examined consumption level is numbered, the better are the products provided a family with a fixed income. Regional consumption pattern differences as well as family size and composition are taken into account by means of correction factors.

9. Foreign trade is regarded as a set of channels for additional product exchange intended for plan improvement. The capacities of these channels are limited and defined by export-import capabilities. The exchange factors for the channels depend on export-import price ratios. Foreign trade is balanced; some limited deficit is allowed.

10. One of the principal characteristics of the proposed model is the way of representing dynamic relationships by a special formulation of a static problem. An optimal plan for a certain *time* is assumed to be adequate if, before and after that time, the state of every particular unit of the national economy at any given time is defined by a certain dynamic growth law. Only the type of such a law is established, and an attempt is made to find its parameters while solving the problem. One may assume, for instance, linear, exponential, or other laws of development. The nature of the law assumed depends on the type of the activity or industry considered, on its age and history, on scarcity of natural resources used, etc. If, for example, we prefer an exponential growth law for an industry, this law is described by a relationship of the type:

$$X^r_{jt} = X^r_{j1}(1 + K^r_j)^{t-1} ,$$

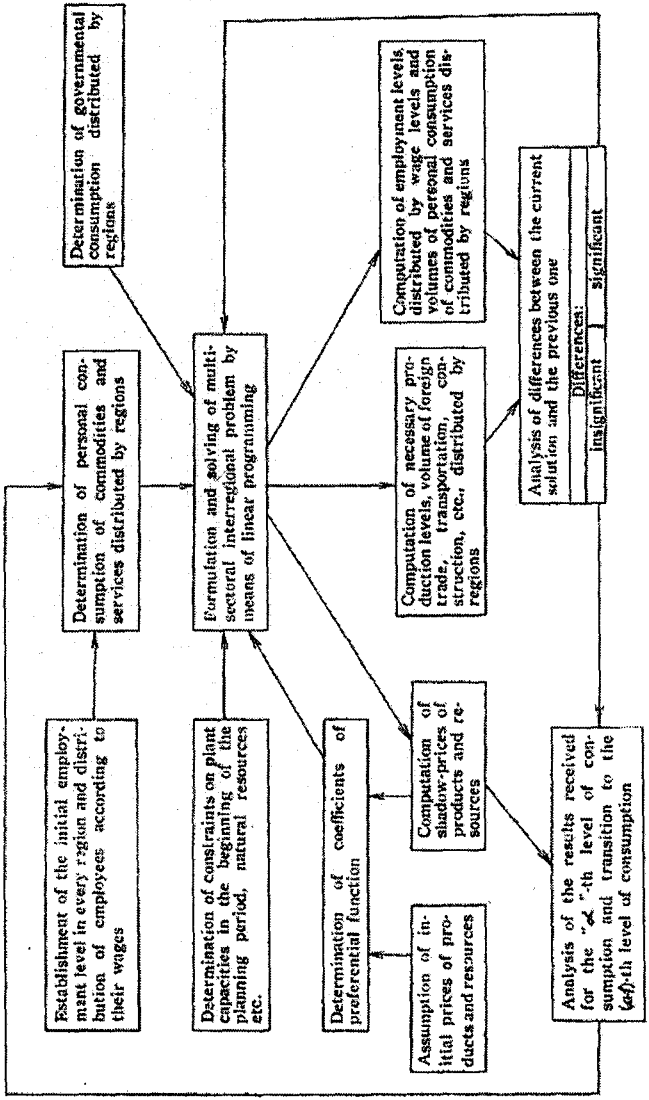
Determination of governmental consumption distributed by regions
Determination of personal consumption of commodities and services distributed by regions
Establishment of the initial employment level in every region and distribution of employees according to their wages
Determination of constraints on plant capacities in the beginning of the planning period, natural resources etc.
Determination of coefficients of preferential function
Assumption of initial prices of products and resources
Formulation and solving of multisectoral interregional problem by means of linear programming
Computation of employment levels, distributed by wage levels and volumes of personal consumption of commodities and services distributed by regions
Computation of necessary production levels, volume of foreign trade, transportation, construction, etc., distributed by regions
Computation of shadow-prices of products and resources
Analysis of differences between the current solution and the previous one
Differences:
insignificant
significant
Analysis of the results received for the "α"-th level of consumption and transition to the (α+1)-th level of consumption

FIGURE 1.

where X_{jt}^{r} = the production level of the jth activity in the rth region at the tth year.

K_j^r = the growth rate for the enterprise.

Solving the problem for the T_0th year we get the value of X_{jt}^{r}. Knowing X_j^r, it is not difficult to calculate the value of the control variable K_j^r. Then, extrapolating the law of growth for T_1 years after the T_0th year, we can determine the required increments of the rated production capacities to maintain a rate of growth K_j^r during this period of time. In turn, knowing the relative distribution of unit capital expense in productive form by the years of the construction period, we can determine every product expenditure at the T_0th year for the necessary construction. In this way dynamic relationships and investment lags are taken into account. Essentially, these are boundary conditions for the T_0th year, if one can assume that after this year the rates and structure of the planning period growth are maintained.

This method of approximate approach to dynamics allows one to better represent spatial and other important aspects of national economy development while keeping within the bounds of a soluble problem size.

11. The optimality criterion of economy growth is formulated in the problem in the following way: the maximum level of personal consumption in the T_0th year is sought, which makes it possible to meet governmental needs, current requirements of production and transportation, and demands of capital investment determined in accordance with item 10.

For the solution of a large-size mathematical programming problem with special structures, we propose an approximate iterative algorithm based on combining a generalized transportation network linear program with a method of inverse matrix formation by expansion in the basic matrix power series. In the latter method, we substitute matrix operations for a large part of optimizing operations. A prototype of this method used for solving simple interregional problems is described in [2]. By means of such an algorithm, which allows one to solve problems of rather large size, one determines the existence of a feasible plan for every consumption level examined in turn. The maximum level of consumption is found for which such a plan still exists. This plan is the problem solution. The aggregated computational flow-chart of the algorithm is given in Figure 1.

The mathematical model of the problem is described in [1]. At the present time, data (some of which are very rough estimates) are collected to find the 1970 year plan for the USSR economy by 16 industries and 5 aggregated regions (zones). The algorithm is being programmed for a computer.

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